WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-III Examination, 2020

MATHEMATICS

PAPER-MTMA-VII

Time Allotted: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Answer any *one* question from the following $5\times1 = 5$

1. (a) If $\vec{\nabla}\times\vec{F}=\vec{0}$, prove that $\vec{F}$ is conservative. 5

(b) Apply Green's theorem to show that the area bounded by a closed curve C is $\oint_C (x\,dy - y\,dx)$. Hence obtain the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 5

(c) Verify Stokes' theorem for $\vec{A}=(2x-y)\hat{i}-yz^2\hat{j}-y^2z\,\hat{k}$, where S is the upper half surface of the sphere $x^2+y^2+z^2=1$, C is its boundary and R is the projection of S on the xy-plane. 5

(d) Verify the divergence theorem for $\vec{A}=4x\hat{i}-2y^2\hat{j}+z^2\,\hat{k}$, taken over the region bounded by $x^2+y^2=4$, $z=0$ and $z=3$. 5

GROUP-B

(ANALYTICAL STATICS)

Answer any *two* questions from the following $7.5\times2 = 15$

2. Show that three coplanar forces P, Q, R acting at the points A, B, C are in astatic equilibrium if they meet at a point on the circumcircle of the triangle ABC and if $P:Q:R=a:b:c$, where a, b, c are the sides of the triangle ABC.

3. Find the centre of gravity of the area of the cardiode $r=a(1+\cos\theta)$.

4. A rough wire which has the shape of an ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, is placed with its x-axis vertical and y-axis horizontal. If μ be the coefficient of friction, find the depth below the highest point of the position of limiting equilibrium of a bead which rests on the wire.

5. What is the energy test of stability? Establish it for a rigid body with one degree of freedom in equilibrium under conservative forces.

6. Deduce the conditions of equilibrium of a system of forces acting on a rigid body from the principle of virtual work.

7. A quadrilateral ABCD is formed by four uniform rods and are freely jointed to each other at their ends. The rods AB, AD being equal and also the rods BC, CD, is freely suspended from the point A. A string is jointed to A and C and is such that ABC is a right angle. Applying the principle of virtual work, show that the tension of the string is $(W+W')\sin^2\theta+W'$, where W is the weight of an upper rod and W' is that of a lower rod and 2θ is equal to the angle BAD.

8. Two forces act, one along the line $y=0,\ z=0$ and the other along the line $x=0,\ z=c$. As the forces vary, show that the surface generated by the axis of their equivalent wrench is $(x^2+y^2)z=cy^2$.

9. Three forces each equal to P, act on a body one at the point $(a, 0, 0)$ parallel to OY, second at the point $(0, b, 0)$ parallel to OZ and the third at the point $(0, 0, c)$ parallel to OX, the axes being rectangular. Show that the central axis passes through the point $\left(-\dfrac{a+2b+3c}{3}, -\dfrac{b+2c+3a}{3}, -\dfrac{c+2a+3b}{3}\right)$.

10. A thin hemispherical bowl of radius b and weight W rests in equilibrium on the highest point of a fixed sphere of radius 'a' which is rough enough to prevent any sliding. Inside the bowl is placed a small smooth sphere w. Show that the equilibrium is not stable unless $w<W\left(\dfrac{a-b}{2b}\right)$.

GROUP-C

(RIGID DYNAMICS)

Answer any *one* question from the following 15×1 = 15

11.(a) Find the moment of inertia of a truncated cone of mass M about its axis, the radius of its ends being a and b. 7

(b) A wire is in the form of a semi-circle of radius a. Show that at an end of its diameter the principal axes in its plane are inclined to the diameter at angles $\frac{1}{2}\tan^{-1}\frac{4}{\pi}$ and $\left(\frac{\pi}{2}+\frac{1}{2}\tan^{-1}\frac{4}{\pi}\right)$. 8

12.(a) Find the expression for moment of momentum about the origin of a rigid body moving in two dimensions. 7

(b) A uniform sphere of radius a, is rotating about a horizontal diameter with angular velocity ω and is gently placed on a rough plane which is inclined at an angle α to the horizontal, the sense of rotation being such as to tend to cause the sphere to move up the plane along the line of greatest slope. Show that if the coefficient of friction be $\tan\alpha$, the centre of the sphere will remain at rest for a time $\dfrac{2a\omega}{5g\sin\alpha}$ and will then move downwards with acceleration $\frac{5}{7}g\sin\alpha$. 8

13.(a) An elliptic lamina can rotate about a horizontal axis passing through a focus and perpendicular to its plane. If the eccentricity of the ellipse be $\sqrt{2/5}$, prove that the centre of oscillation will be at the other focus. 7

(b) A homogeneous sphere rolls down an imperfectly rough fixed sphere, starting from rest at the highest point. If the spheres separate when the line joining their centres makes an angle θ with the vertical, prove that $\cos\theta + 2\mu\sin\theta = Ae^{2\mu\theta}$, where A is the function of μ only. 8

GROUP-D

(HYDROSTATICS)

SECTION-I

Answer any *one* question from the following $10\times1 = 10$

14.(a) From a semi-circle of radius a whose diameter is in the surface of a liquid, a circle is cut out, whose diameter is the vertical radius of the semi circle. Show that the depth of the centre of pressure of the remainder is $\dfrac{9\pi a}{8(16-3\pi)}$. 5

(b) A hollow cone whose weight is half that of the water it can contain, floats in water in stable equilibrium with its axis vertical and vertex downwards. Prove that if α be the semi-vertical angle of the cone, then $\cos^6\alpha < \dfrac{729}{1024}$. 5

15.(a) For a body floating freely in a homogeneous fluid at rest under gravity, prove that $HM = \dfrac{Ak^2}{V}$, where the symbols have their usual meanings. 5

(b) A gaseous atmosphere in equilibrium under gravity has its pressure p, density ρ and absolute temperature T connected by $p = k\rho\gamma = R\rho T$, where k, γ, R are constants. Show that the temperature decreases upwards at a constant rate. 5

SECTION-II

Answer any *one* question from the following $5\times1=5$

16.(a) A gas satisfying Boyle's law $p=k\rho$ is acted on by forces $X=-\dfrac{y}{x^2+y^2}$, $Y=\dfrac{x}{x^2+y^2}$. 5

Show that the density varies as $e^{\theta/k}$, where $\tan\theta=\dfrac{y}{x}$.

(b) Prove that the surface of separation of two liquids of different densities, which do not mix, at rest under gravity is a horizontal plane. 5

(c) Prove that if the forces per unit mass at (x, y, z) parallel to the axes are $y(a-z)$, $x(a-z)$, xy, the surfaces of equal pressure are hyperbolic paraboloids. 5

(d) A liquid fills the lower half of a circular tube of radius 'a' in a vertical plane. If the tube is now rotated about the vertical diameter with angular velocity ω such that the liquid is about to separate into two parts, then show that $\omega=\sqrt{\dfrac{2g}{a}}$. 5

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